

Neutron stars in a universe with a cosmological constant

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Summary

- We numerically integrated stellar structure equations with correction for a cosmological constant term Λ for a given neutron star equation of state using Euler's method.
- A negative cosmological constant reduces the maximum mass and radius of a star upon comparison with the $\Lambda = 0$ values.

Motivation

A well-known explanation for the expansion rate of the universe is the existence of an all-pervading cosmological constant. [1] From a particle physicist's point of view, the cosmological constant can be interpreted as the energy density of the vacuum. This interpretation poses a problem because the value of the cosmological constant from cosmological observations differ from the theoretically predicted value of quantum field theory by 120 orders of magnitude. This discrepancy is the so-called cosmological constant problem, which is still an open problem to this day. [2]

The cosmological constant is known to play an important role in the formation and evolution of structures in the universe. Recently, Silbar et al. suggested the investigation of the effect of the cosmological constant to the structure of neutron stars. [4] Bordbar et al. then calculated the effect of different values of the cosmological constant to a neutron star with a chosen equation of state (EOS). [5] They found that an upper bound for the mass exists for a positive cosmological constant. To this day, the exact equation of state of the neutron star is an active area of investigation. This affords us a leeway to investigate on whether a similar bound exists for a neutron star of a different equation of state.

In this study, we will calculate the effect of the cosmological constant to the mass-radius relation for a neutron star equation of state as described in Silbar et al.

Numerical Implementation

The Tolman-Oppenheimer-Volkov equations for a neutron star with correction for a cosmological constant Λ is given by [3] as

$$\frac{d\bar{p}}{dr} = -\frac{\alpha\bar{\epsilon}\bar{m}}{r^2}\left(1+\frac{\bar{p}}{\bar{\epsilon}}\right)\left(1+\frac{\beta r^3\bar{p}}{\bar{m}}-\frac{\Lambda r^3}{2R_0\bar{m}c^2}\right)\left(1-\frac{2\alpha\bar{m}}{r}\right)^{-1} \quad (1)$$

$$\frac{d\bar{m}}{dr} = \beta r^2\bar{\epsilon} \quad (2)$$

where $p = \epsilon_0\bar{p}$ and $\epsilon = \epsilon_0\bar{\epsilon}$ are pressure and energy density, respectively. These have been rewritten in terms of dimensionless variables $\bar{p}, \bar{m}$ and $\bar{\epsilon}$. This set of coupled ODEs is numerically integrated via Euler's method for the EOS of a non-interacting neutron Fermi gas

$$\bar{\epsilon}(\bar{p}(r)) = A_{NR}\bar{p}^{3/5} + A_R\bar{p} \quad (3)$$

where

$$A_{NR} = 2.24216 \qquad A_R = 2.8663 \quad (4)$$

In this EOS, the non-relativistic term dominates at low pressures while the relativistic term dominates at high pressures. This EOS holds for non-relativistic ($k_F \ll m_n$) and relativistic case ($k_F \gg m_n$). Integration using Euler's method requires the evaluation of Eqn. (1) at the origin where it is singular. To cope with the singularity, we used an asymptotic series solution for $\bar{p}, \bar{m}$ of the form

$$\bar{p}(r) = p_0 + c_1 r + c_2 r^2 + c_3 r^3 + \dots \quad (5)$$

$$\bar{m}(r) = r^\sigma(d_0 r + d_1 r + d_2 r^2 + \dots) \quad (6)$$

and then solved for the coefficients. We used these expressions to evaluate at points near the origin. To check for consistency with literature [3], we calculated the mass-radius relation for a central dimensionless pressure of 0.03, $\alpha = R_0 = 1.476km$, $\beta = 0.03778$, $\Lambda = 0$. Our result agrees with the literature mass-radius relation which has a maximum value $M \approx 0.8M_{sun}$ at $R \approx 11km$.

Results

The current estimated value of the cosmological constant is $\Lambda \approx 10^{-52}$ m^{-2}, which is negligible when compared to the denominator $\approx 10^{14}$. Thus the effect of this value to the mass-radius relation does not differ from when $\Lambda = 0$. This gives us a choice on the value of the parameter. For some values of the cosmological constant, $\Lambda \gtrsim 10^8$ the pressure increases without bound due to the change in sign of the second correction factor in Eq. (1). Putting in values smaller than $\Lambda \approx 10^8$, we observed that the pressure reduces to zero at some final radius. This corresponds to the surface of the neutron star and the enclosed mass at this radius is the total mass.

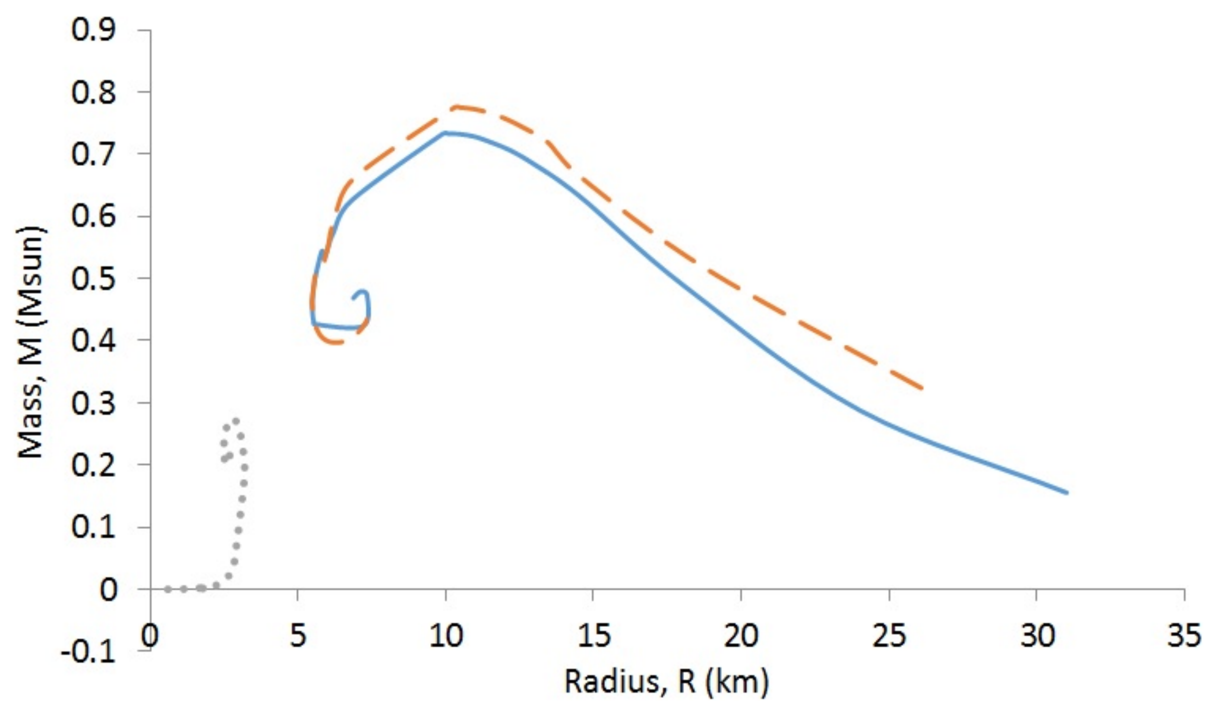

Fig. 1. Mass-radius relation for $\Lambda = 0$ case (blue continuous line), $\Lambda = 10^6$ (orange dashed line), and $\Lambda = -10^{10}$ (gray dotted line)

For negative values of the cosmological constant (anti-de Sitter case) the maximum mass and radius of the neutron star were reduced as seen in Fig. 1. These results show that certain values of the cosmological constant have an effect on the mass-radius relation of neutron stars.

References

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